Technical Notes

TECHNICAL NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed 6 manuscript pages and 3 figures; a page of text may be substituted for a figure and vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Supersonic Jet Noise Suppression by Coaxial Cold/Heated Jet Flows

D.S. Dosanjh,* P.K. Bhutiani,† and K.K. Ahuja‡
Syracuse University, Syracuse, N.Y.

Introduction

THE noise radiated by a single round supersonic (high specific thrust), turbulent, cold or heated jet is inherently too intense. Its reduction is possible if, instead, multistream jet flows of complex geometry are employed whereby the supersonic jet or exhaust flows are suitably changed so as to modify, weaken, shield, and/or eliminate the more dominant jet noise sources. This can be achieved if 1) the spatial extent of the supersonic region of the jet flow is shortened; 2) the shear of the exhaust flow just downstream of the nozzle is reduced; 3) the noise-generation effectiveness of the mixing regions is diminished, and 4) the repetitive shock structure in the under- or overexpanded jet flows is weakened or eliminated. Since these flow modifications ought to be affected with minimal thrust penalty, the noise-suppression achieved by inserting mechanical devices in the high-speed exhaust flow is therefore counter productive. However, the use of coflowing interacting coaxial high-speed jet flows issuing from a two-nozzle configuration of suitable geometry, design, and operating conditions, first reported by Dosanjh et al.,[1] embodies many of the preferred attributes of an effective supersonic jet noise-suppression approach. In these investigations, the mode of operation of coaxial supersonic jets differed from what is found to be effective for noise suppression from coaxial subsonic jets[2] in that, to optimize noise reductions, the outer (annular) underexpanded cold jet was instead maintained at a comparatively higher pressure ratio (i.e., at a higher jet flow Mach number or velocity) than that of the inner (round) cold-jet flow. These supersonic jet noise-suppression studies were extended[3,4] to cold/heated coaxial jet where either the inner or outer jet could be heated. More recently, coaxial heated jets operated in such an "inverted" mode have been investigated for their possible noise-suppression applications in duct-burning turbo-fan[5] and advanced variable cycle turbo-jet[6,7] engines.

Coaxial Nozzles and Procedure for Acoustic Data Acquisition

These investigations were conducted with a coaxial two-nozzle configuration with coplanar exits, where the diameter of the inner (round) convergent nozzle was 1 in. As it was found to be advantageous in earlier "inverted" mode studies of coaxial jets,[4] the exit areas of the outer (annular) and inner convergent nozzles were selected to be nearly equal ($A_{e2}/A_{e1} \doteq 1.1$). Therefore, the width of the annular nozzle = 0.216 in. The inner nozzle exit had a finite (but small) lip thickness with the ratio of the lip thickness to the inner nozzle radius $\doteq 0.06$.

Operating stagnation pressure ratios [$\xi = P_R + P_a/P_a$, where P_R is the reservoir pressure (psig) and P_a is the ambient pressure, ranging 1-4] and the reservoir temperatures T_R of the inner and the outer nozzles ranging 70-1000°F, were controlled independently. The nozzle configuration was operated either 1) as a single inner (round) or outer (annular), cold or heated jet; or 2) as coaxial cold/heated jets in three different modes of operation. These operational modes are: cold-cold (both jets unheated; $T_{R1} = T_{R2}$); cold-heated (with the annular jet heated and the inner jet operated cold; $T_{R2} > T_{R1}$); and heated-cold (with inner jet heated and the outer jet operated cold; $T_{R1} > T_{R2}$).

Third-octave sound pressure level were recorded at eight azimuthal positions, 15 deg apart, from 15 to 120 deg (with respect to the jet axis) on an arc of 10-ft radius in a 26×21×14-ft anechoic chamber and were corrected for microphone pressure response, free-field effects, and atmospheric absorption. OASPL's, 1/3-octave PWL's, and OAPWL's were calculated. Here only a few salient experimental results are briefly noted. For detailed results, see Refs. 3 and 4.

Experimental Results and Discussion

Noise from Single Round and Annular Supersonic Heated Jets

The individual round and annular jets were operated at the same pressure ratio, $\xi \doteq 3.04$, and reservoir temperature, $T_R = 600°F$ (i.e., $V_j/a_0 = 1.66$ where V_j is the fully-expanded flow velocity and a_0 is the ambient acoustic speed). At the mid to higher frequencies, PWL's of the annular jet are lower than those of the round jet (Fig. 1). For the round heated jet, the shock-associated discrete or narrow band noise is quite strong, but it is not that dominant for the annular heated jet. OASPL's of the annular heated jet were 5 to 10 dB lower than those of the round heated jet, with the difference increasing toward higher angles which at 90 deg was about 7 dB.[4] At higher temperatures ($T_R = 850°F$) and $\xi_2 \doteq 3.04$, similar behavior of PWL's and OASPL's was observed. Since the single annular and round jets of the coaxial configuration, with coplanar exits and nearly the same exit areas, were operated at the same ξ and T_R (i.e., at the same fully-expanded jet-flow velocity), it may be concluded that the

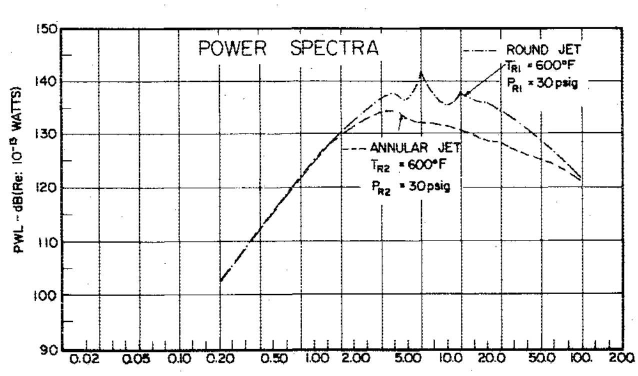

Fig. 1 Radiated noise from single round and annular supersonic heated jets. $P_R = 30$ psig or $\xi_2 \doteq 3.04$; $T_R = 600°F$; $V_j/a_0 \doteq 1.66$; exit area ratio $A_{e2}/A_{e1} \doteq 1.1$.

Presented as Paper 76-507 at the AIAA 3rd Aero-Acoustics Conference, Palo Alto, Calif., July 20-23, 1976; submitted Nov. 16, 1976; revision received Oct. 4, 1977.

Index categories: Noise; Supersonic and Hypersonic Flow.

*Professor, Mechanical and Aerospace Engineering. Associate Fellow AIAA.
†Presently at General Electric, Cincinnati, Ohio. Member AIAA.
‡Presently at Lockheed-Georgia, Marietta, Georgia. Member AIAA.

annular convergent underexpanded jet is inherently quieter than an equivalent round convergent underexpanded jet. This advantage of the annular jet stems chiefly from its more rapid decay to sonic speeds (repetitive shock structure disappears) over a shorter axial distance downstream of the nozzle exit than that for the equivalent round jet.[4]

Minimum-Noise Conditions for Cold/Heated Coaxial Jets

The variation of the logarithm of the acoustic efficiency η [radiated acoustic power W/mechanical power of the jet $T^2/\dot{m}$, (T=total thrust, $\dot{m}$=total mass flow rate of coaxial jets)] with the inner jet pressure ratio ξ_1 (keeping the annular jet-pressure ratio ξ_2 fixed) is shown in Fig. 2 for the three modes of operation of the coaxial cold/heated jets. For the cold-cold and the cold-heated modes of operation, with increasing ξ_1, the acoustic efficiency decreases to a minimum value. The set of operating reservoir pressures (ξ_2, ξ_1) and temperatures (T_{R2}, T_{R1}) which correspond to the least acoustic efficiency for a particular mode of operation of coaxial jets of a given coaxial nozzle geometry, size and configuration is designated as the minimum-noise condition. For the cold-heated mode of operation, with fixed $P_{R2}=30$ psig or $\xi_2 \doteq 3.04$, $T_{R2} \doteq 600°F$, and $T_{R1}=70°F$, the minimum-noise condition occurred for the inner jet-reservoir pressure $P_{R1} \doteq 15$ psig or $\xi_1 \doteq 2.02$, with only minor variation in the acoustic efficiency for $P_{R1}=12$-18 psig. The corresponding ratio of the outer to inner fully-expanded flow velocities $V_{j2}/V_{j1} \doteq 1.73$ and for $A_{e2}/A_{e1}=1.1$, $\dot{m}_2/\dot{m}_1=1.17$. Moreover, the ratio of the acoustic efficiency η_c of the coaxial cold/heated jets at the minimum-noise condition to the acoustic efficiency η_2 of the annular heated jet operated alone $\doteq 0.5$. This means that by such use of the coaxial jets, the radiated noise from the single, noise-wise dominant, annular heated jet has been reduced by 50%. For the cold-cold mode of operation, at the minimum-noise operating pressure ratios $\xi_2=3.04$, $\xi_1 \doteq 2.02$; $V_{j2}/V_{j1} \doteq 1.22$; $\dot{m}_2/\dot{m}_1 \doteq 1.65$; and $\eta_c/\eta_2=0.36$. Similar behavior of the acoustic efficiency for the cold-heated mode is also observed when the annular jet is heated to higher reservoir temperatures (for $T_{R2}=850°F$, $V_{j2}/V_{j1} \doteq 1.89$), except that for the same ξ_2 noise levels (SPL's, PWL's and OAPWL's) for the higher T_{R2}'s and therefore higher V_{j2}'s were correspondingly higher.[3,4] From the comparison of the acoustic efficiency of the cold-cold and the cold-heated modes of operation of coaxial jets it may be concluded that with the pressure ratio ξ_2 of the annular jet kept fixed (such that the annular jet is underexpanded), the minimum noise is attained when the inner round jet pressure ratio ξ_1 is such that the round jet is also weakly underexpanded, where $\xi_2 > \xi_1$, $T_{R2} > T_{R1}$, and $V_{j2}/V_{j1} > 1$. The observed noise reductions from such an "inverted" mode of operation of coaxial jets have been shown[4] to depend on 1) the outer annular jet of higher specific thrust being quieter than an equivalent round jet; 2) the reduction of shear of the noise-wise dominant annular jet flow just downstream of the nozzle exit owing to the presence of the inner jet flow; 3) the repetitive shock structure and jet flow modifications; and 4) the decay of the coaxial jet flows to sonic speeds in a shorter distance downstream of the nozzle exit than that for an equivalent single round jet.

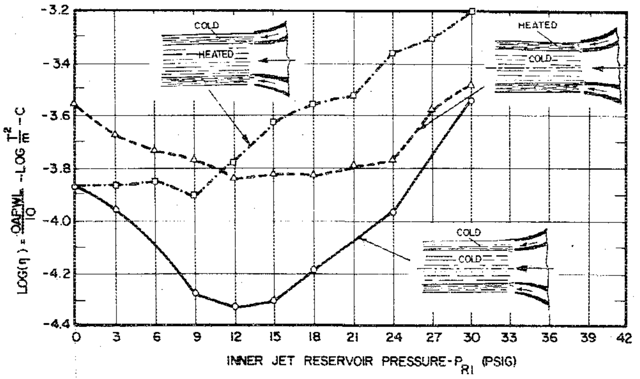

Fig. 2 Variation of acoustic efficiency of coaxial cold/heated jets with inner jet reservoir pressure. Outer jet reservior pressure fixed at $P_{R1}=30$ psig ($\xi_2 \doteq 3.04$); cold-heated mode $T_{R1}=70°F$, $T_{R2}=600°F$, $V_{j2}/a_0 \doteq 1.66$; heated-cold mode $T_{R1}=600°F$, $T_{R2}=70°F$, $V_{j2}/a_0 \doteq 1.17$; cold-cold mode $T_{R1}=T_{R2}=70°F$, $V_{j2}/V_{j1} \doteq 1.22$.

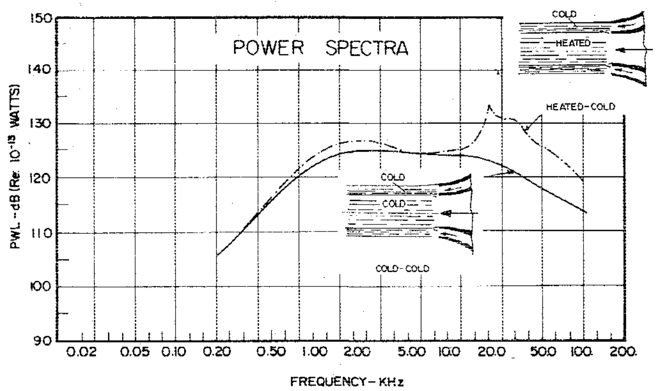

Fig. 3 Acoustic power spectra of coaxial jets (cold-cold and heated-cold modes; same mean flow velocities but different reservior pressures).

P_{R1}, psig	P_{R2}, psig	T_{R1}, °F	T_{R2}, °F	V_{j1}/a_0	V_{j2}/a_0	V_{j2}/V_{j1}	
15	30	70	70	0.96	1.17	1.22	heated-cold
6	30	600	70	0.96	1.17	1.22	cold-heated

Role of Operating Pressure and Velocity Ratios

For the heated-cold mode of operation of the coaxial jets, a definitive minimum-noise condition was not observed (Fig. 2). The inner round heated jet flow with $T_{R1} > T_{R2}$ becomes the dominant source of noise at lower inner jet reservoir pressures P_{R1}, resulting in a higher acoustic efficiency for the heated-cold mode than the cold-heated mode. This crossover in the acoustic efficiency for the heated-cold mode of operation with $\xi_2 \doteq 3.04$, $T_{R2}=70°F$ and $T_{R1}=600°F$, occurs for $P_{R1} \geq 10$ psig where $V_{j2}/V_{j1} \leq 1$.

However, when for the heated-cold mode of operation ($\xi_2 \doteq 3.04$, $T_{R2}=70°F$, and $T_{R1}=600°F$) the operating pressure ratio ξ_1 is so selected that the fully expanded jet flow velocities of the individual annular and round jets and their ratio $V_{j2}/V_{j1} > 1$ are the same as those for their cold-cold mode of operation at the minimum-noise condition ($\xi_2 \doteq 3.04$, $\xi_1 \doteq 2.02$, and $T_{R2}=T_{R1}=70°F$), PWL's of the heated-cold mode at higher frequencies are substantially (8-10 dB) higher than those for cold-cold mode at the minimum-noise condition (Fig. 3). This indicates that the shock and flow modifications for the heated-cold mode of operation (because of lower ξ_1) are not as effective as for the cold-cold mode at the minimum-noise condition, even though at the respective individual inner and outer jet flow velocities, the mean flow-velocity jump at the interface of the jets and $V_{j2}/V_{j1} > 1$ are maintained the same. Therefore, for optimizing noise reductions from coaxial jets operated in the "inverted" mode, maintaining the velocity ratio $V_{j2}/V_{j1} > 1$, is not, by itself, sufficient.

For coaxial supersonic jets of a given configuration operated in the "inverted" mode at given outer (annular) jet-pressure ratio ξ_2, it is therefore necessary that the operating pressure ratio ξ_1 ought to be selected first for minimum-noise condition with $\xi_2 > \xi_1$. Then the operating temperatures T_{R1} and T_{R2} of the inner and outer jet, respectively, should be tailored (maintaining $T_{R2} > T_{R1}$) to obtain the desired individual fully expanded flow velocities and velocity ratios $V_{j2}/V_{j1} > 1$ such that the noise reductions are optimized.

Acknowledgment

This Note is based on research supported by the Department of Transportation, Grant No. DOT-OS-20094.

References

[1] Dosanjh, D. S., Yu, J. C, and Abdelhamid, A. N., "Reduction of Noise from Supersonic Jet Flows," *AIAA Journal*, Vol. 9, Dec. 1971, pp. 2346-2453.

[2] Williams, T. J., Ali, M. R. M., and Anderson, J. S., "Noise and Flow Characteristics of Coaxial Jets," *Journal of Mechanical Engineering Science*, Vol. 11, 1969, pp. 132-134.

[3] Dosanjh, D. S., Bhutiani, P. K., and Ahuja, K. K., "Supersonic Jet Noise Suppression by Coaxial Cold/Heated Jet Flows," AIAA Paper 76-507, Palo Alto, Calif., July 1976.

[4] Dosanjh, D. S., Bhutiani, P. K., and Ahuja, K. K., "Supersonic Jet Noise Suppression by Coaxial Multi-Nozzle Cold/Heated Jet Flows," Mechanical and Aerospace Engineering Department, Syracuse University, Final Report submitted to DOT, March 1977.

[5] Kozlowski, Hilary, "Coannular Nozzle Noise Characteristics and Applications to Advanced Supersonic Transport Engines," *Proceedings of the SCAR Conference*, Part 2, NASA CP-001, Nov. 1976, pp. 491-504.

[6] Lee, Robert, "Coannular Plug Nozzle Noise Reduction and Impact on Exhaust System Design," *Proceedings of the SCAR Conference*, NASA CP-001, Nov. 1976, pp. 505-524.

[7] Guiterrez, Orlando, A., "Aeroacoustic Studies of Coannular Nozzles Suitable for Supersonic Cruise Aircraft Applications," *Proceedings of the SCAR Conference*, NASA CP-001, Nov. 1976, pp. 471-490.

Frequencies of Annular Plate and Curved Beam Elements

J. F. Wilson* and D. P. Garg†
Duke University, Durham, N.C.

I. Introduction

THERE is a precedent for modeling horizontally-curved highway bridges as curved beams[1,2] even though such bridges are generally composite structures consisting of curved girders and plates or box sections. For relatively lightweight vehicles, one can visualize a guideway cross section with a width perhaps ten times the depth dimension, especially for a two-lane elevated span. For such sections, plate theory may seem more appropriate for dynamic analysis than beam theory. This concern motivated the calculation and comparison of the free vibration frequency spectra of curved beam and annular plate elements. Limits of both theories are discussed in terms of dimensionless frequency parameters.

II. Curved Plate Free Vibrations

Based on classical theory for a uniform plate of stiffness D, deflection w, and mass density per unit surface area ρ, the governing homogeneous equation is[3]

$$D\nabla^4 w + \rho w_{,tt} = 0 \tag{1}$$

where the Laplacian operator ∇^2 is written in terms of the polar coordinates (r,θ) of Fig. 1.

Presented as Paper 77-371 at the AIAA/ASME 18th Structures, Structural Dynamics, and Materials Conference, San Diego, Calif., March 21-23, 1977; submitted March 31, 1977; revision received Oct. 31, 1977.

Index category: Vibration.

*Associate Professor, School of Engineering.
†Professor, School of Engineering.

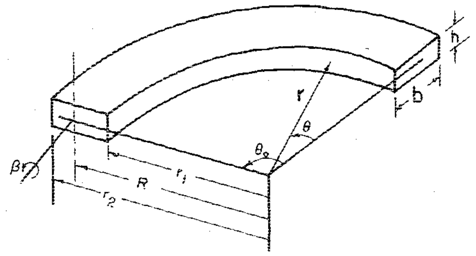

Fig. 1 Horizontally curved beam or plate segment.

Assuming this annular plate segment undergoes free harmonic vibrations at frequencies q, or

$$w(r,\theta,t) = W(r,\phi)\sin qt \tag{2}$$

then Eq. (1) reduces to

$$\nabla^2 W - k^4 W = 0 \qquad k^4 = \rho q^2/D \tag{3}$$

The mode solutions to Eq. (3) are chosen in the form

$$W(r,\theta) = \sum_{n=1}^{\infty}\Big[A_n J_n(kr) + B_n Y_n(kr) + C_n I_n(kr) + D_n K_n(kr)\Big]\sin(n\pi\theta/\theta_0) \tag{4}$$

where J_n and Y_n are Bessel functions of the first and second kind, respectively; I_n and K_n are modified Bessel functions of the first and second kind, respectively; and $A_n,\ldots,D_n$ are constants. Equation (4), with Eq. (2), satisfies both the zero deflection conditions at the end supports, $w(r,0,t) = w(r,\theta_0,t) = 0$ and the conditions of zero bending moment, $M_\theta(r,0,t) = M_\theta(r,\theta_0,t) = 0$. It remains to satisfy the conditions of zero radial moment and shear reaction at the two radial boundaries $r = r_i$ $(i=1,2)$ or

$$M_r(r_i,\theta,t) = 0 \qquad V_r(r_i,\theta,t) = 0 \tag{5}$$

When Eqs. (5) are evaluated according to Eqs. (2) and (4), the result is four homogeneous, algebraic equations of the form

$$A[A_n B_n C_n D_n]^T = 0 \tag{6}$$

Nontrivial solutions to Eq. (6) exist only if the determinate of the coefficients vanishes, or

$$\det|A| = 0 \tag{7}$$

where the elements of the A matrix for $i = 1,2$ are

$$a_{ij} = S_n''(\alpha_i\lambda) + \frac{\nu}{\alpha_i\lambda}S_n'(\alpha_i\lambda) - \frac{\nu\beta_n^2}{(\alpha_i\lambda)^2}S_n(\alpha_i\lambda) \tag{8a}$$

$$\begin{aligned} a_{i+2,j} = {} & S_n'''(\alpha_i\lambda) + \frac{1}{\alpha_i\lambda}S_n''(\alpha_i\lambda) - \frac{1}{(\alpha_i\lambda)^2}S_n'(\alpha_i\lambda) \\ & + \frac{2\beta_n}{(\alpha_i\lambda)^3}S(\alpha_i\lambda) + \frac{(1-\nu)}{(\alpha_i\lambda)^3}\beta_n^2 S(\alpha_i\lambda) \\ & - \frac{\beta_n^2}{(\alpha_i\lambda)^2}S_n'(\alpha_i\lambda) - \frac{(1-\nu)}{(\alpha_i\lambda)^2}\beta_n^2 S_n'(\alpha_i\lambda) \end{aligned} \tag{8b}$$

In Eqs. (8), the symbol S_n is interpreted by J_n, Y_n, I_n, and K_n for $j = 1,2,3,4$, respectively. For convenience, the arguments (kr_i) of the Bessel functions were replaced by separate